A modelling framework for estimating benefits of increasing on- and off-farm irrigation efficiency

A training manual on how
to build an optimising model
for an irrigation area

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NATIONAL PROGRAM FOR
Sustainable Irrigation

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Introduction

Irrigation efficiency can be increased both on- and off-farm. The use of irrigation water can be reduced on farm, through the adoption of water saving technologies and efficient water management practices and through changes in farm activities and by reusing runoff water. The seepage and evaporation losses of water from dilapidated open canals off-farm can be reduced by relining them or eliminated by replacing them with pipes. This paper first discusses the issues in modelling irrigation systems and then presents a generic modelling framework which can be adapted to most of the irrigation systems to estimate the impacts of increasing both on- and off-farm irrigation efficiency.

Issues in modelling

An irrigation system model designed to estimate the impacts of increased irrigation efficiency should have both off- and on-farm components linked together and is typically build around a canal network, which keeps track of water flow after conveyance losses and off takes from farms at different nodes are accounted for. The water is diverted to the system from the River or a main delivery canal. The on-farm component comprises a number of farms linked together through the delivery network. For each farm, for each land type, agronomically feasible set of cropping activities and for each cropping activity a set of feasible water application methods need to be specified with relevant input output coefficients and price and cost parameters. Given that the multitude of decision-making farm units and their interaction are to be included, the model can easily grow into a large system if careful thought is not given at the design stage. The level of detail of the system to be included in the model depends on the size of the study area, the degree of homogeneity in farming systems between farms located at different points in the system and the inflow, storage, depletion and outflow characteristics of the system water balance.

For a system as large and complex as the Murrumbidgee Irrigation Area and District (MIA &D) system, it is practically impossible to include the microcosm of the whole system. The MIA and district system has a canal network totalling 1200 kilometres in length supplying water to over 2000 farms. This system's water balance is characterised by a number of inflows including reuse of surface runoff, canal escape and some deep percolated water in addition to diversions from the river and an in-line storage (the Barren Box Swamp). The aggregation of farms and the associated supply network to divisions is one of the possible solutions to reduce the overall model to a manageable size while retaining a reasonable amount of details of the diverse farming systems adopted in different areas. On the other hand, microcosmic details may be included in a system model designed for a smaller area. For example, the model, which ABARE developed for the Stanbridge system within the MIA, has all the farms and the entire deliver network represented in detail. The Stanbridge system is situated in Leeton within the Murrumbidgee Irrigation Area and comprises 67 canal reaches of variable lengths totalling 18 kilometres. The water drawn from the Gogaldrie branch canal of the MIA system is supplied to a total of 47 farms.

The size of the model depends not only on the number of farms or groups of farms in the model, but on the range of farming systems and the inflow, storage and outflow characteristics of the irrigation system. In addition to contributing to the size of the model, the latter factor also adds to the complexity of the model. In order to keep the model simple and of manageable size, in addition to the aggregation option discussed earlier, the modeller may also have to look into areas where trade offs can be made between the inclusion of details leading to a large model and those of a complex model. In the Stanbridge model, only a simplified representation of the farming system was included while the microcosm of the delivery system and all 47 farms were included as we were more interested in the cost of delivering water to farms at different nodes of the system. Compared to this, in the MIA model we aggregated the farms and the delivery network in to 26 divisions but the farming system in each division is modelled in detail. The canal network represented in the MIA model included the Main canal and the major branch canals supplying water to the division perimeters while the canals found within each division are represented with a simple conveyance loss rate.

The time step assumed in water balancing also affects the size of the model. The net irrigation requirements of crops on a given day depend on the residual soil moisture, rainfall, temperature and relative humidity on that day. Due to day to day variability in the factors affecting net irrigation requirements, the time step included in the model should be short enough to adequately capture such variability. Inclusion of a daily time step becomes necessary, if precision irrigation techniques such as deficit irrigation and irrigation sequencing are to be included as alternative water saving options in order to capture the benefits of trade offs in intra seasonal water allocation.

If such details are to be included as alluded to before, some avenues for trade off between size and complexity must be found to keep the model to a manageable size. In both the Stanbridge and MIA models we assumed that the farmers strive to obtain the maximum yield and thus irrigate to meet the potential evapotranspiration requirement after rainfall and capilary rise are netted out. In these models which have been formulated on an annual basis, the water balancing was done on an average per day basis for each month within the year.

In addition to the above physical aspects, the model should also include the institutional arrangements employed in pricing and delivery of water by the water authority. Currently, irrigation water charging is based on the principle of full cost recovery in a region. Normally this takes the form of uniform pricing, where the total cost of delivering water to all farms in the region is shared equally between all farmers, by charging all farmers the same price. The irrigation authority delivers water following a roster system to make the best use of the existing infrastructure capacity at its disposal so that any cost due to constraints in these capacities are equally shared by the farmers in the system. This neglects the differences in delivery costs associated with the spatial distribution of farms in the region. For example, farms located closer to the head reaches are likely to have a lower delivery cost than farms further downstream. The differences in delivery cost can be large particularly during peak months and in drier years when water value increases due to constraints in the infrastructure capacities and the volume of water allocated, respectively. The infrastructure capacity can be binding for only those farmers in some irrigation reaches, but the costs imposed by these constraints are shared equally between all the farms mainly through the operation of the roster system which is designed to deliver each farm of its entitlement.

Oftentimes, economic model solutions characterise economically efficient outcomes, which even though desirable from a societal point of view may be different from what could practically be achieved. The criterion of maximising the systemwide profits in a standard primal model of an irrigation system unavoidably leads to model solutions characterised by an efficient outcome which means that each individual farm is charged the actual cost of delivery of water to the farm including the cost of infrastructure constraints. However, in reality delivery costs and capacities are shared between farms with some farms paying more and others less than the actual cost. In order, to overcome this problem in the standard primal model and to adequately represent the institutional arrangements in the model, the modeller has to first derive the conceptually difficult dual formulation (the mirror image) of the (primal) model.

Then the modeller has to combine relevant primal (volumes) and dual (price) variables together to include an additional terms in the criterion. As this additional term is a quadratic term derived by multiplying one variable with another, simplex LP algorithm can no longer be used and modeller has to switch to a non linear algorithm. The primal-dual formulation would unavoidably almost double the size of the model and the complexity may more than double as both volume and price variables now work in tandem. A model designed in this manner even for a small area will be large and complex and a spreadsheet based optimising package such as *What's Best* may no longer be able to provide an optimal solution. We went through this process and ended up developing two versions of the model for each of the Stanbridge and MIA systems: an efficient pricing version and a uniform pricing version where the costs of delivery and infrastructure capacities are shared. Our models were coded in GAMS (General Algebraic Modelling System). The uniform pricing version was finally used in the estimation of benefits from increasing irrigation efficiency.

Regardless of the size of the study area, every effort should be made to keep the model least complicated and to a manageable size. This is best done by representing in the model, after aggregating at an appropriate level, only those components in detail, which are essential in the context of the system to be modelled and more importantly the particular issues to be investigated. The desirability of such a modelling strategy and the differences between irrigation systems mean, separate models need to be developed for different irrigation systems. Even though there are advantages in developing a generic model to be used for most of the irrigation systems, such a model would be unavoidably too large and complex as the model need to be able to accommodate the full range of diversity in farming and irrigation systems observed in a reasonably large area such as the Murray Darling Basin.

A simple model

Whether it is a simple or complicated model the general concepts, structure, important components, data requirements and types of model outputs are the same. Therefore, this training manual will focus on a simple model with a view of giving the trainees a good understanding and an appreciation of various aspects involved in developing similar models for irrigation systems in their states or regions. The simple model will be used to explain, step by step, the process of building a model for an irrigation system. Both algebraic and tableau formulations of the same model are developed. The model represented in an LP tableau can be run with spreadsheet based optimising packages such as *What's Best* while an algebraic representation will be suitable for larger and more complicated models so that they can be easily coded in GAMS. This training course, by using the both algebraic and Tableau formulations, will cover in detail only the efficient pricing version of the model. However, an algebraic representation of the uniform pricing version of the simple model will discussed briefly.

In the model, the canal network is represented in terms of 5 sequential reaches separated by nodes with farm groups located along the channel reaches, however, for simplicity there is assumed to be one farm group at each reach.

A bird's eye view

Before getting into details, a general overview of the model is discussed in this section just to give a flavour of the model. The simple model has *r* farm groups and an equal number of reaches of variable length, μ_r, each separated by a node $i \neq j$. For each farm group, *r*, the model includes 5 sets of activities (columns) and 5 sets of constraints (rows) (table 1). For each reach, *r,* the activity sets (columns) included in table 1 from right to left are (1) refurbishment options (2) runoff reuse option (3) cropping options by application technologies (4) peak daily flow rate by refurbishment option and (5) annual flow by refurbishment option. For each reach, *r*, the constraint sets (rows) included in table 1 from top to bottom are (1) decision control on refurbishment options (2) an upper bound on the peak daily flow by refurbishment option (3) an upper bound on annual flow by refurbishment option (4) a systemwide water accounting identity for annual flow and (5) a systemwide water accounting identity for the peak flow. Each systemwide water accounting identity tracks water flow along the system, by accounting for each reach the inflow from the upstream reach (from the river or the weir for the first reach), crop use, conveyance losses and the outflow to all the downstream reaches. The systemwide water accounting identities form the linchpin of the model as they link the farms to the off-farm delivery system. Note that the water balancing is done on an annual basis in order to keep the model small compared to an average per day basis for each month as in the MIA model. However, water balancing in the peak month is included as the peak flow rate is related to the cost of refurbishment investment and any constraint on the peak flow rate can be binding. The cell entries in table 1 are the input output coefficients which will be described in the next section. The far right column or the right hand side (RHS) of the inequality sign gives the level specified for each constraint. The objective function of the model (first row of table 1) is specified to represent, for the whole irrigation system, the annual gross margin on all farms less the annualised cost of investment in infrastructure refurbishment, the annualised cost of investment in reuse dams, the annual cost of pumping water from or to the dam and the annual cost of water delivery to farm offtakes. The model solutions are obtained by maximising the objective function.

Table 1 A simplified structure of the model

		Reach 1					Reach 2					Reach 3					Reach 4					Reach 5						
		Refur invest	Dam water	Crop by ap tech	Peak flow	An flow	Refur invest	Dam water	Crop by ap tech	Peak flow	An flow	Refur invest	Dam water	Crop by ap tech	Peak flow	An flow	Refur invest	Dam water	Crop by ap tech	Peak flow	An flow	Refur invest	Dam water	Crop by ap tech	Peak flow	An flow		RHS
	Objective function	$-\alpha_k \cdot \mu_r$	$-P^d_r$	M_{rct}	$-\beta_k \cdot \mu_r$	$-P$	$-\alpha_k \cdot \mu_r$	$-P^d_r$	M_{rct}	$-\beta_k \cdot \mu_r$	$-P$	$-\alpha_k \cdot \mu_r$	$-P^d_r$	M_{rct}	$-\beta_k \cdot \mu$	$-P$	$-\alpha_k \cdot \mu_r$	$-P^d_r$	M_{rct}	$-\beta_k \cdot \mu_r$	$-P$	$-\alpha_k \cdot \mu_r$	$-P^d_r$	M_{rct}	$-\beta_k \cdot \mu_r$	$-P$		
R1	Conveyance	1																									<=	1
R1	Peak flow control				1																						<=	CK_r
R1	Annual flow control					1																					<=	F_r
R1	Divert					1																					<=	D_r
R1	Sys accnt (annual)		-1	$\varepsilon_{ct}/\theta_{ct}$		$-\sigma$					1																<=	0
R1	Sys accnt(peak)		$-\pi$	γ_{ct}/θ_{ct}	$-\sigma$					1																	<=	0
R2	Conveyance						1																				<=	1
R2	Peak flow control									1																	<=	CK_r
R2	Annual flow control										1																<=	F_r
R2	Sys accnt (annual)							-1	$\varepsilon_{ct}/\theta_{ct}$		$-\sigma$					1											<=	0
R2	Sys accnt(peak)							$-\pi$	γ_{ct}/θ_{ct}	$-\sigma$					1												<=	0
R3	Conveyance											1															<=	1
R3	Peak flow control														1												<=	CK_r
R3	Annual flow control															1											<=	F_r
R3	Sys accnt (annual)												-1	$\varepsilon_{ct}/\theta_{ct}$		$-\sigma$					1						<=	0
R3	Sys accnt(peak)												$-\pi$	γ_{ct}/θ_{ct}	$-\sigma$					1							<=	0
R4	Conveyance																1										<=	1
R4	Peak flow control																			1							<=	CK_r
R4	Flow control																				1						<=	F_r
R4	Sys accnt (annual)																	-1	$\varepsilon_{ct}/\theta_{ct}$		$-\sigma$					1	<=	0
R4	Sys accnt(peak)																	$-\pi$	γ_{ct}/θ_{ct}	$-\sigma$					1		<=	0
R5	Conveyance																					1					<=	1
R5	Peak flow control																								1		<=	CK_r
R5	Annual flow control																									1	<=	F_r
R5	Sys cont (annual)																						-1	$\varepsilon_{ct}/\theta_{ct}$		$-\sigma$	<=	0
R5	Sys cont (peak)																						$-\pi$	γ_{ct}/θ_{ct}	$-\sigma$		<=	0

Investment in refurbishment

For practical purposes, individual reaches or channel segments including more than one contiguous reach are often considered as the smallest divisible units for investment in refurbishment of infrastructure. For a given smallest divisible unit of investment, the decision to invest involves selecting one of the *k* technically feasible options (relining with clay, concrete or membrane or replacing with pipes). In this training manual, individual reaches are considered as the smallest divisible units.

$$\sum_{k} I_{rk} \leq 1 \qquad (1)$$

For each reach, *r*, the decision to invest in one of the *k* refurbishment options is handled by the use of a binary variable, I_{rk} which can take a value of either one or zero and the requirement that for each *r* only one refurbishment option must be chosen.

For each reach, the decision to invest on a particular refurbishment option depends on the cost and the incremental benefits of the water saved to all the farms in the area. The increased benefits of water saved depend on the reduction in conveyance losses due to refurbishment and the options available on farms to use saved water. For each reach, *r*, the cost of refurbishment depends on the size of the channel, which should be large enough to carry the peak demand flow rate and the length of the reach (μ_r). For each refurbishment option, *k*, the cost of investment can be estimated, once the relationship between the peak flow rate, q_k and the cost of refurbishment per lineal metre, RC_k is known. Such relationships can be estimated using actual data based on past refurbishment activities and or by taking an engineering approach based on hydraulic relationships between flow rates and channel or pipe geometry (Appendix A). For each refurbishment option, *k*, a simple linear relationship between the annualised investment cost per lineal metre, RC_k and the peak flow rate, q_k can be expressed as $RC_k = \alpha_k + \beta_k . q_k$, where, α_k and, β_k are coefficients which can be estimated by linear regression by using data on annualised cost of refurbishment per lineal metre at different peak flow rates. In the absence of actual data, *pseudo* data on cost per lineal metre at different peak flow rates can be developed by simulating the hydraulic relationship between the peak flow rate and the canal/pipe geometry (Appendix A).

The total annualised cost of investment in refurbishment can be represented in the objective function as

$$\sum_{r,k}\alpha_k \mu_r I_{rk} + \sum_{r,k}\beta_k \mu_r q_{rk}.$$

Note that, the decision variable on investment, I_{rk}, is included in the first term but not in the second term, as the variable q_{rk} cannot be multiplied by another variable in LP. In order to make sure, that the refurbishment option, *k*, in q_{rk} is the same as the refurbishment option, *k* in I_{rk} the following relationship is included in the model.

$$q_{rk} \leq CK_r I_{rk} \tag{2}$$

For each reach, *r*, and each refurbishment option, *k*, the peak flow, q_{rk} cannot exceed a constant, CK_r times the decision variable on investing in infrastructure, I_{rk}. In this manner, the *k* in q_{rk} is forced to be the same as the *k* in I_{rk}. The constant CK_r should be set at a sufficiently high level so that it has zero value (not binding). The equation 2 can also be used to represent an effective upper bound on the peak flow rate, if there is a limit to the size of the channel or pipe to be constructed. In that case, this constraint would have a non-zero value that means the objective value can be improved by relaxing this constraint.

Just as for the peak flow, in order to make sure, that the refurbishment option, *k*, in annual flow, Q_{rk} is also the same as the refurbishment option, *k* in I_{rk} the following relationship is also included in the model.

$$Q_{rk} \leq FC_r I_{rk} \tag{3}$$

For each reach, *r*, and each refurbishment option, *k*, the annual flow, Q_{rk} cannot exceed constant, FC_r times the decision variable on investing in infrastructure. In this manner, the *k* in Q_{rk} is forced to be the same as the *k* in I_{rk}. The constant FC_r should be set at a sufficiently high level so that it has zero value (not binding).

Table 2 shows how these relationships can be incorporated in the LP tableau. The relationships 1-3 along with the expression representing the cost of investment cover the cost side of the refurbishment investment decision in the model and the manner in which these relationships can be represented in the LP tableau is given in the first 5 rows of table 2.

Table 2 Investment in refurbishment								
	Invest Conv Earth	**Invest Conv Conc**	**Invest Conv Piped**	**Peak flow earth**	**Peak flow conc**	**Peak flow piped**		**RHS**
Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$	$-\beta^e.\mu_r$	$-\beta^c.\mu_r$	$-\beta^p.\mu_r$		
Conveyance type	1	1	1				<=	1
Peak flow control -earth	$-CK_r$			1			<=	0
Peak flow control -conc		$-CK_r$			1		<=	0
Peak flow control -pipe			$-CK_r$			1	<=	0
Sys accnt (peak)				$-(1-\delta^e.\mu_r)$	$-(1-\delta^c.\mu_r)$	$-(1-\delta^p.\mu_r)$		

Systemwide water accounting – peak flow

In order to keep a track of peak daily water flow at successive nodes a systemwide water accounting identity needs to be introduced (equation 4).

$$\sum_{c.t}\frac{\gamma_c}{\theta_{ct}}A_{rct}+\sum_k q_k^{jr'}-\sum_k q_k^{ir}(1-\delta_{kr}l_r)-\pi DW_r \le 0 \qquad (3)$$

For farm r located at reach r, the sum over crops c and application technologies t, the peak daily net (after rainfall and capillary rise) evapotranspiration requirement, γ_c, adjusted for application efficiency, θ_{ct}, plus the water flow to the next downstream node $\sum_{k'} q_{k'}^{jr'}$ cannot be less than the peak daily water flow from the supplying node i to that reach with all the refurbishment options k, q_{rk} (note only one refurbishment option is chosen) adjusted for daily conveyance loss in that reach occurring at rate, $(1-\delta_{kr}\mu_r)$ plus π portion of the annual volume of runoff water reused after storage, DW_r. For simplicity it is assumed that the proportion of the annual volume of water reused is given. The modeller should exercise judgement in selecting a value for π depending on how constraining the water allocation and the delivery infrastructure at the peak month. As mentioned before, the systemwide water accounting identity links the farms through the off-farm delivery system. The manner in which the systemwide water accounting identity for the peak flow is represented in the LP tableau is presented in full in Appendix B and in parts in tables 2-4. For each reach, r, the conveyance loss in the peak month can be presented as in the bottom row of table 2, crop water requirement in table 3 and the demand by the next reach in table 4. Note that the system continuity equation for reach 1 extends to reach 2 (table 4) and reach 2 to reach 3 and so on (Appendix B). As the peak demand will be in the summer (either in December or January), the demand for winter crops such as wheat in the peak month can be assumed zero.

Table 3 Crop water demand in the peak month									
	Dam water	**Vine furrow**	**Vine drip**	**Citrus furrow**	**Citrus drip**	**Rice tech1**	**Rice tech2**	**Wheat tech1**	**Wheat tech2**
Objective function	$-P^d_r$	MVF_r	MVD_r	MCF_r	MCD_r	MRT_{1r}	MRT_{1r}	MWT_{1r}	MWT_{1r}
Sys accnt (peak)	$-\pi$	γ^{vf}/θ^{vf}	γ^{vd}/θ^{vd}	γ^{cf}/θ^{cf}	γ^{cd}/θ^{cd}	γ^{r1}/θ^{r1}	γ^{r2}/θ^{r2}		

Table 4 Water accounting between reach 1 and reach 2 - peak flow									
					Reach 2				
	Invest Conv Earth	**Invest Conv Conc**	**Invest Conv Piped**	**Dam water**	**Peak flow earth**	**Peak flow conc**	**Peak flow piped**		**RHS**
Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$	$-P^d_r$	$-\beta^e.\mu_r$	$-\beta^c.\mu_r$	$-\beta^p.\mu_r$		
R1 Sys accnt (peak) (R1)					**1**	**1**	**1**	<=	0
R2 Conveyance type	1	1	1					<=	1
R2 Peak flow control -earth	$-CK_r$				1			<=	0
R2 Peak flow control -conc		$-CK_r$				1		<=	0
R2 Peak flow control -pipe			$-CK_r$				1	<=	0
R2 Sys accnt (peak)				$-\pi$	$-(1-\delta^e.\mu_r)$	$-(1-\delta^c.\mu_r)$	$-(1-\delta^p.\mu_r)$		

Net irrigation requirement

For each farm group, *r*, the net irrigation requirement is derived after netting out rainfall, capillary rise and reuse of runoff water from the total evapotranspiration requirement.

$$\sum_{c.t}\frac{\varepsilon_c}{\theta_{ct}}A_{rct} - DW_r - CW_r \leq 0 \tag{4}$$

For farm *r* located at reach *r*, the sum over crops *c* and application technologies *t,* the annual net (after rainfall and capillary rise) evapotranspiration requirement, ε_c, adjusted for application efficiency, θ_{ct} cannot exceed the annual offtake of irrigation water, CW_r plus the annual volume of runoff water reused after storage, DW_r.

Systemwide water accounting for the annual flow

Just as for the peak flow, a systemwide water accounting identity is also introduced for annual flow to keep a track of the annual water flow at successive nodes (equation 5). The annual flow rate in the head reach cannot exceed the annual demand for all the uses of water downstream which include, farm off takes and conveyance losses from the channels.

$$CW_r + \sum_k Q_k^{jr'} - \sum_k Q_k^{ir}(1-\delta_{kr} l_r) \leq 0 \qquad (5)$$

For farm *r* located at reach *r*, the annual offtake by the farm, CW_r plus the water flow to the next downstream node $\sum_{k'} Q_{k'}^{jr'}$ cannot be less than the annual water flow from the supplying node *i* to that reach with all the refurbishment options *k* (note only one refurbishment option is chosen as per equation 1), Q_{rk} adjusted for daily conveyance loss in that reach occurring at rate, $(1-\delta_{kr}\mu_r)$. The manner in which the systemwide water accounting identity for annual flow is represented in the LP tableau is presented in full in Appendix B and in parts in tables 5-7. The annual flow rates and conveyance losses are accounted for in the last row of table 5, the use of runoff water and crop water requirements in the last row of table 6 and the demand by the next reach in the second and the last rows of table 7. Note that the systemwide water accounting identity for the annual flow for reach 1 extends to reach 2 and reach 2 to reach 3 and so on. Unlike in the case of peak flow, the water requirement of winter crops such as wheat is included in the systemwide water accounting identity for the annual flow.

Table 5 Annual diversion at source, channel flow and conveyance losses

	Invest Conv Earth	Invest Conv Conc	Invest Conv Piped	Annual flow earth	Annual flow conc	Annual flow piped		RHS
Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$					
R1 Conveyance type	1	1	1				<=	1
R1 Flow control -earth	$-F_r$			1			<=	0
R1 Flow control -conc		$-F_r$			1		<=	0
R1 Flow control -piped			$-F_r$			1	<=	0
R1 Divert -earth				1			<=	*D*
R1 Divert -conc					1		<=	*D*
R1 Divert -piped						1	<=	*D*
R1 Sys accnt (annual)				$-(1-\delta^e.\mu_r)$	$-(1-\delta^c.\mu_r)$	$-(1-\delta^p.\mu_r)$		

Diversion

The water is diverted from a main canal (source) at the Weir to the uppermost reach of the branch canal.

$$\sum_k Q_{1k} \leq D \qquad (7)$$

The sum of annual water flows from node 1 to reach 1 with all the refurbishment options (note only one of the three options is chosen as per equation 1) cannot exceed the annual allocation of water for all the farms in the system, *D*.

Table 6 Water application to crops and reuse of runoff water on an annual basis													
	Invest Dam	Dam water	Irrig water	Vine furrow	Vine drip	Citrus furrow	Citrus drip	Rice tech1	Rice tech2	Wheat tech1	Wheat tech2		RHS
Objective function	$-ID_r$	$-P^d_r$	$-P^i_r$	MVF_r	MVD_r	MCF_r	MCD_r	MRT_{1r}	MRT_{1r}	MWT_{1r}	MWT_{1r}		
Run off		1		σ^{vf}	σ^{vd}	σ^{cf}	σ^{cd}	σ^{r1}	σ^{r2}	σ^{w1}	σ^{w2}	<=	0
Irrigation			-1	$\varepsilon^{vf}/\theta^{vf}$	$\varepsilon^{vd}/\theta^{vd}$	$\varepsilon^{cf}/\theta^{cf}$	$\varepsilon^{cd}/\theta^{cd}$	$\varepsilon^{r1}/\theta^{r1}$	$\varepsilon^{r2}/\theta^{r2}$	$\varepsilon^{w1}/\theta^{w1}$	$\varepsilon^{w2}/\theta^{w2}$	<=	1
Dam water	$-DK_r$	1										<=	0
Sys accnt (annual)		-1	1										

Table 7 Continuity in annual flow between adjoining reaches								
	Reach 2							
	Invest Conv Earth	Invest Conv Conc	Invest Conv Piped	Annual flow earth	Annual flow conc	Annual flow piped		RHS
Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$					
R1 Sys accnt (annual) (R1)				**1**	**1**	**1**	<=	0
R2 Conveyance type	1	1	1				<=	1
R2 Flow control -earth	$-F_r$			1			<=	0
R2 Flow control -conc		$-F_r$			1		<=	0
R2 Flow control -piped			$-F_r$			1	<=	0
R2 Sys accnt (annual)				$-(1-\delta^e.\mu_r)$	$-(1-\delta^c.\mu_r)$	$-(1-\delta^p.\mu_r)$		

Investment in reuse dam

$$ID_r \leq 1 \tag{8}$$

For each reach, r, the decision to invest in a reuse dam of given capacity is handled by the use of a binary variable, ID_r which can take a value of either one or zero. The decision to invest on a reuse dam depends on the cost and the incremental benefits of the water saved to the farm. For simplicity, we have assumed that the size of the dam has already been decided[1].

[1] Alternatively we may assume that decisions have to be made on both the optimal size of the dam and whether or not to invest. In that case the equation 8 could be modified as given in equation 8a.

$$\sum_s ID_{rs} \leq 1 \tag{8a}$$

For each reach, r, the decision to invest in a reuse dam of size s is again handled by the use of a binary variable, ID_{rs}, which can take a value of either one or zero and the requirement that for each reach r only one dam must be chosen.

$$DW_r - DK_r * ID_r \leq 0 \qquad (9)$$

Assuming that the size of the dam is given, in each farm *r*, the annual volume of runoff water stored and reused cannot exceed the assumed capacity of the storage, DK_r times the value (either 1 or 0) of the decision variable ID_r[2].

$$DW_r - \sum_{c.t} \sigma_{ct} A_{rct} \leq 0 \qquad (10)$$

In each farm *r*, the annual volume of runoff water stored and reused cannot exceed the sum of annual irrigation and rainfall runoff water produced, σ_{ct}, from all the irrigated land, A_{rct}.

The annualised cost of investment and the cost of pumping water to or from the dam depending on where it is to be located on the farm should be debited from the objective function as $ID_r * DCST_r$ and $P_r^d * DW_r$, respectively.

The manner in which the relationships (8), (9) and (10) and the costs of reuse of run off water can be represented in the LP tableau is given in table 6.

Cost of delivering water

The cost of delivering water to all the farm groups should be debited from the objective function. Denoting, the delivery charge per ML as P_r, the total cost of delivering water can be expressed as $\sum_r CW_r P_r$. For each farm, *r*, the delivery cost of water is represented in the first row of the LP tableau (see table 6). It is assumed that the delivery charge is calculated on the volume of water received at the farm offtake, and the unit delivery charge is uniform across farms thereby ignoring the difference in the cost of water delivery between upstream and downstream farms. The unit delivery charge may be assumed to have set by including a component to cover the average cost of water delivery including water lost in transmission (per ML of water delivered to all farms).

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[2] If the size of the dam is also a decision variable, then the equation 9 can be modified as in equation 9a.

$$DW_r - \sum_s DK_{rs} * ID_{rs} \leq 0 \qquad (9a)$$

The difference between the delivery charge and the price of water need to be understood. If the annual flow control constraint, F_r and the peak flow control constraint , CK_r for each reach and the allocation constraint for the whole system, D are set at sufficiently high level so that these constraints are not binding then the price of water equals the delivery charge as all three constraints have zero values. If the water allocation for the whole system is binding then the price of water is greater than the delivery charge. The difference between the price of water and the delivery charge in this case represents the value of the allocation constraint. In the model, trading of water between farms occurs but trading of water with outside the system does not occur. If trading of water with outside the system is included, then the value of the allocation constraint will be equal to the opportunity cost of water outside the system. Trading of water outside the system is included in the MIA model.

Allocation of land

For each reach, *r,* land available can be allocated between crops and for each crop between alternative application technologies.

$$\sum_{b\in c,t} A_{rbt} \leq IBLND_r - LT_r \tag{11}$$

In each reach *r*, the sum of the land used for all irrigated broadacre crops *b* grown with all application technologies, *t*, cannot exceed the given area of irrigated broadacre land, $IBLND_r$, less the area of land transferred out, LT_r, to plant dry crops.

$$\sum_{t} A_{rt}^{rice} \leq RLIM_r \tag{12}$$

In each reach *r*, the area of irrigated broadacre land planted to rice with all application technologies, *t*, cannot exceed the hydraulic loading (environmental) limit, $RLIM_r$.

$$A_r^{dry} \leq DBLND_r + LT_r \tag{13}$$

In each reach *r*, the area used for dry broadacre crops cannot exceed the given area of dry broadacre land, $DBLND_r$, plus the area of irrigated land transferred in, LT_r, to plant dry crops.

$$\sum_{t} A_{rht} \leq HLND_{rh} \tag{14}$$

In each reach *r* and for each horticulture crop stand, *h*, the sum of the land used for all application technologies *t* cannot exceed the total area of that crop, $HLND_{rh}$.

The land allocation relationships 11 to 14 are represented in the LP tableau as shown in table 8.

Table 8 Allocation of land between crops and application technologies												
	Vine furrow	Vine drip	Citrus furrow	Citrus drip	Rice tech1	Rice tech2	Wheat tech1	Wheat tech2	Land trans	Dry cropin		RHS
Objective function	MVF_r	MVD_r	MCF_r	MCD_r	MRT_{1r}	MRT_{1r}	MWT_{1r}	MWT_{1r}		MDW_r		
Vine land	1	1									<=	$HLND_{vr}$
Citrus land			1	1							<=	$HLND_{cr}$
Irrig land					1	1	1	1	1		<=	$IBLND_r$
Rice lim					1	1					<=	$RLIM_r$
Dry land									-1	1	<=	$DBLND_r$

The annual gross margins earned on all farms are given in the following expression and are credited to the objective function as also shown in table 8.

$$\sum_{rct} MIB_{ct} * IBA_{rct} + \sum_{r} MDB * DBA_r + \sum_{rct} MH_{ct} * HA_{rct}$$

Solving the model

The simple model is developed as a tableau in *What's Best* optimising software package which is called within *Excel* as an *add in*. The tableau for the 5 farm groups included in the system has 88 rows and 110 columns (Appendix B). There are twenty binary variables represented in 20 of these columns. Even though, the dimension of the matrix (88*110) indicates a small model, the inclusion of 20 binary variables makes it somewhat a large model when it comes to finding a solution.

The solution to the model is obtained by maximising

$$\begin{aligned} &\sum_{rbt} MIB_{bt} \times A_{rbt} + \sum_{r} MDB \times A_r^{dry} + \sum_{rht} MH_{ht} \times A_{rht} \\ &- \sum_{r,k} \alpha_k \mu_r I_{rk} + \sum_{r,k} \beta_k \mu_r q_k - ID_r * DCST_r - P_r^d * DW_r - \sum_{r} CW_r P_r \end{aligned} \qquad (15)$$

with respect to nonnegative values of variables subject to inequality constraints for conditions (1) – (14). The objective function in (15) represents, for the whole irrigation system, the annual gross margin on all farms less the annualised cost of investment in infrastructure refurbishment, the annualised cost of investment in reuse dams, the annual cost of pumping water from or to the dam and the annual cost of water delivery to farm offtakes. The objective function is represented in the first row of the model (Appendix B).

Notation

Subscripts, superscripts and ranges

i and j	node	i,j =1, ..., 67
r and r'	reach, division assigned	r,r' =1, ..., 67
c	all crops	c= citrus, vines, rice, irrig wheat, dry wheat
b	irrigated broadacre crops	b= rice (r) and irrig wheat (w)
h	horticulture crops	h= citrus (o) and vines (v)
t	irrigation technology for	t = flood intensity 1 (t1), flood intensisity 2 (t2) broad furrow (f) and drip (d)
d	dam water	
k	refurbishment option	k= earth (e), concrete (c) piped (p)

Variables

Q_k^{ir}	annual water flow from node i to reach r with refurbishment option k (ML/year)
q_{rk}	peak water flow from node i to reach r with refurbishment option k (ML/day)
A_{rct}	area planted to crop cn with application technology t in farm group r (Ha)
DW_r	volume of dam water used by farm group r (ML/year)
CW_r	volume of irrigation water used by farm group r (ML/year)
I_{rk}	the decision to invest in refurbishment option k in reach r (0 or 1).
ID_r	the decision to invest in reuse of runoff water in reach r (0 or 1).
LT_r	transfer of irrigated broadacre land for dry cropping (Ha)

Parameters

P	delivery charge of water ($/ML)
P_r^d	cost of pumping water from or to the reuse dam ($/ML)

$DCST_r$	Annualised cost of storage in farm group r ($/year)
M_{ct}	gross margin of crop *c* planted with application technology *t* ($/ha)
μ_r	length of reach *r* (metres)
δ_{kr}	proportion of the flow rate lost due to evaporation and seepage with refurbishment option *k* along reach *r* per metre
λ_c	peak net evapotranspiration requirement of crop *c* (ML/ha/day)
ε_c	annual net evapotranspiration requirement of crop *c* (ML/ha/year)
σ_{ct}	irrigation and rainfall water runoff from crop *c* planted with application technology *t*
θ_{ct}	irrigation application efficiency of crop c*n* planted with application technology *t*
D	Annual allocation of water (ML/year)
CK_r	Upper bound on the peak flow rate in reach *r* (ML/day)
FC_r	Upper bound on the annual flow for reach *r* (ML/year)
DK_r	limit on drawing water from the dam in region farm *r* (ML/year)
π_r	proportion of the runoff water stored in the dam reused in the peak month in farm *r* (ML/day)
$IBLND_r$	area of irrigated broadacre land available to farm group *r* (ha)
$DBLND_r$	area of dry broadacre land available to farm group *r* (ha)
$RLIM_r$	limit on the rice area for farm group *r* (ha)
$HLND_{rh}$	area of land planted with horticulture crop *h* in farm group *r* (ha)
α_k	intercept on the refurbishment cost equation ($/metre/year)
β_k	slope coefficient on the refurbishment cost equation ($/metre/Mlof peak flow/year)
$DCST_r$	cost of investment in runoff reuse dam in farm group *r* ($/year)

Uniform pricing formulation of the simple model

The efficient pricing version of the model discussed in the preceding section represents the conditions for optimal behaviour by farmers as well as the water authority within a well functioning water market. In particular, water authorities in this model are assumed to charge a price that reflects the cost of delivering water, including conveyance losses, to each farm. A uniform pricing version of the same model is formulated in this section to represent conditions for optimal behaviour by farmers as well as the water authority, but subject to a uniform water price prevailing regardless of the difference between farms in costs of conveyance losses and infrastructure capacities. Uniform pricing of irrigation water entails some economic losses and consequently this form of pricing is not economically efficient (Hafi, Klijn and Toyne, 1999). The uniform pricing model is formulated by trading off some efficiency elements in the criteria of the efficient pricing model to achieve equity in the form of uniform pricing. Model solutions provide the optimal price of water, the allocation of water between farm groups the optimal allocation of land and water between alternative cropping activities and for each cropping activity the optimal mix of water application technologies. The model also solves for prices of resources, which are measured in annual rent equivalents.

In order to adequately represent the institutional arrangements for uniform pricing in the model, the mirror image or the dual formulation of the primal (efficient pricing) model was derived. The equations in the primal model represent the volume conditions whereas the equations in the dual formulation represent the price conditions for the efficient pricing formulation. The price conditions derived in this manner for both the peak water flow and annual water flow are then modified to represent uniform pricing currently practiced in irrigation areas. The value of water lost in delivering water to all the farm groups is estimated by multiplying the volumes of water lost by the price of water and then credited to the objective function.

In addition to the doubling of the size and more than doubling of the complexity of the model, some of the features of the primal formulation which come handy particularly in efficiency studies have also been sacrificed. The binary variables come handy when different options of investments in channel refurbishment and reuse dams are to be modelled due to the lumpiness of such investments. The volume conditions for such investment decisions – for example – equation 1 for refurbishment and equation 8 for reuse dams return a value of either 1 or 0. Unlike in these volume conditions where the decision variable can be explicitly defined to be a binary variable, the decision variable in the corresponding price conditions take implicit values which cannot be numerically defined to be binary variables. Consequently, we have a situation where the binary value returned for the investment decision by the volume condition is not matched by the costate value for that decision implicit in the price returned by the corresponding price condition. As a result, decisions, which involve lumpy investments, cannot be modelled endogenously in the uniform price formulation. An alternative approach is to run the model separately for each investment option and then evaluate the options by comparing the results obtained. As binary variables are not included in the uniform pricing model, the flow control equations (2) and (3) are no longer required. However, a simplified version of the peak flow control equation (2) is included to represent the channel capacity constraint at source. The equations in the efficient pricing formulation are rewritten below as volume conditions for the uniform pricing formulation after dropping the *k* subscript, which earlier represented the refurbishment option. The interpretation of each volume condition is the same as that given for the corresponding equation in the efficient pricing version of the model. Each volume condition is followed by an expression, which defines the value of that condition, denoted by V_r^n where *n* is the equation number. Each rewritten volume condition can be interpreted in the following manner. The component on the left of the less than equal to sign cannot be greater than the component on the right hand side. If the left hand side component is less than the right hand side component, the value of the *n*th condition, V_r^n must be zero.

Volume conditions

$$Q_1 \le D \text{ and } V^{16}(Q_1 - D) = 0 \qquad (16)$$

$$q_{11} \le CK_1 \text{ and } V_1^{17}(q_{11} - CK_1) = 0 \qquad (17)$$

$$\sum_{c.t}\frac{\gamma_c}{\theta_{ct}}A_{rct} + \sum_{jr'} q^{jr'} + \delta_r\mu_r q^{ir}) - \pi DW_r \le q^{ir} \quad \text{and}$$

$$V_r^{18}\left(\sum_{c.t}\frac{\gamma_c}{\theta_{ct}}A_{rct} + \sum_{jr'} q^{jr'} + \delta_r\mu_r q^{ir}) - \pi DW_r - q^{ir}\right) = 0, \text{ for } \forall\ i \text{ and } r \qquad (18)$$

$$\sum_{c.t}\frac{\varepsilon_c}{\theta_{ct}}A_{rct} - DW_r \le CW_r \text{ and } V_r^{19}\left(\sum_{c.t}\frac{\varepsilon_c}{\theta_{ct}}A_{rct} - DW_r - CW_{rr}\right) = 0, \text{ for } \forall\ r \qquad (19)$$

$$CW_r + \sum_{jr'} Q^{jr'} + \delta_r\mu_r Q^{ir}) \le Q^{ir}$$

$$\text{and } V_r^{20}\left(CW_r + \sum_{jr'} Q^{jr'} + \delta_r\mu_r Q^{ir}) - Q^{ir}{}_r\right) = 0, \text{ for } \forall\ i \text{ and } r \qquad (20)$$

$$DW_r \le DK_r \quad \text{and } V_r^{21}\left(DW_r - DK_r\right) = 0, \text{ for } \forall\ r \qquad (21)$$

$$DW_r \le \sum_{c.t} \sigma_{ct} A_{rct} \text{ and } V_r^{22}\left(DW_r - \sum_{c.t} \sigma_{ct} A_{rct} \right) = 0 \text{, for } \forall\, r \quad (22)$$

$$\sum_{b \in c,t} A_{rbt} \le IBLND_r - LT_r \text{ and } V_r^{23}\left(\sum_{b \in c,t} A_{rbt} - \left(IBLND_r - LT_r\right) \right) = 0 \text{, for } \forall\, r \quad (23)$$

$$\sum_t A_{rt}^{rice} \le RLIM_r \text{ and } V_r^{24}\left(\sum_t A_{rt}^{rice} - RLIM \right) = 0 \text{, for } \forall\, r \quad (24)$$

$$A_r^{dry} \le DBLND_r + LT_r \text{ and } V_r^{25}\left(A_r^{dry} - \left(DBLND_r + LT\right)\right) = 0 \text{, for } \forall\, r \quad (25)$$

$$\sum_t A_{rht} \le HLND_{rh} \text{ and } V_r^{26}\left(\sum_t A_{rht} \le HLND_{rh} \right) = 0 \text{, for } \forall\, r \quad (26)$$

Price conditions

$$\frac{\gamma^{ir-wheat}}{\theta_t^{ir-wheat}} V_r^{18} + \frac{\varepsilon^{ir-wheat}}{\theta_t^{ir-wheat}} V_r^{19} - \sigma^{ir-wheat} V_r^{22} + V_r^{23} \ge M_t^{ir-wheat}$$

$$\text{and } A_{rt}^{ir-wheat}\left(\frac{\gamma^{ir-wheat}}{\theta_t^{ir-wheat}} V_r^{18} + \frac{\varepsilon^{ir-wheat}}{\theta_t^{ir-wheat}} V_r^{19} - \sigma^{ir-wheat} V_r^{22} + V_r^{23} - M_t^{ir-wheat} \right) = 0 \text{, for } \forall\, r \text{ and } t \quad (27)$$

In each farm group *r*, for the irrigated wheat crop, for each application technology *t*, on a per hectare basis, the value of the upper bound on the peak flow rate, plus the value of water used over the year, less the value of runoff water produced over the year, plus the value of land, V_t^{23}, cannot be less than the given gross margin for that irrigated wheat crop managed with that water application method, $M_t^{ir-wheat}$. If the sum of these values is greater than the gross margin, then the land is not used for growing irrigated wheat with application technology *t*.

$$\frac{\gamma^{rice}}{\theta_t^{rice}} V_r^{18} + \frac{\varepsilon^{rice}}{\theta_t^{rice}} V_r^{19} - \sigma^{rice} V_r^{22} + V_r^{23} + V_r^{24} \ge M_t^{rice}$$

$$\text{and } A_{rt}^{rice}\left(\frac{\gamma^{rice}}{\theta_t^{rice}} V_r^{18} + \frac{\varepsilon^{rice}}{\theta_t^{rice}} V_r^{19} - \sigma^{rice} V_r^{22} + V_r^{23} + V_r^{24} - M_t^{rice} \right) = 0 \text{, for } \forall r \text{ and } t \quad (28)$$

The price condition for rice is similar to that of irrigated wheat except for an additional term, V_t^{24}, which is the value of the hydraulic loading constraint on the rice area.

$$V_r^{25} \ge M^{dry} \text{ and } A_r^{dry}\left(V_r^{25} - M^{dry} \right) = 0 \text{, for } \forall r \quad (29)$$

In each farm group *r*, for dryland wheat, on a per hectare basis, the value of land, V_t^{26}, cannot be less than the given gross margin for dryland wheat, M^{dry}.

$$\frac{\gamma_h}{\theta_{ht}} V_r^{18} + \frac{\varepsilon_h}{\theta_{ht}} V_r^{19} - \sigma_h V_r^{22} + V_r^{26} \geq M_{ht} \text{ and}$$

$$A_{rht}\left(\frac{\gamma_h}{\theta_{ht}} V_r^{18} + \frac{\varepsilon_h}{\theta_{ht}} V_r^{19} - \sigma_h V_r^{22} + V_r^{26} - M_{ht}\right) = 0 \text{, for } \forall r,\ h \text{ and } t \quad (30)$$

In each farm group r, for each horticultural crop, h, for each application technology t, on a per hectare basis, the value of the upper bound on the peak flow rate, plus the value of water used over the year, less the value of runoff water produced over the year, plus the value of land, V_t^{26}, cannot be less than the given gross margin for that horticulture crop managed with that water application method, M_{ht}. If the sum of these values is greater than the gross margin, then the application technology t will not be adopted on that horticulture crop land.

$$V_r^{23} \geq V_r^{25} \text{ and } LT_r\left(V_r^{23} - V_r^{25}\right) = 0 \quad (31)$$

In each farm group r, on a hectare basis, the value of irrigable broadacre land cannot be less than the value of dry broadacre land, and if the former exceeds the latter, then there will be no irrigable broadacre land planted to dry crops.

$$V_r^{19} - V_r^{22} + \pi V_r^{18} \leq P_r^d + V_r^{21} \text{ and } DW_r\left[V_r^{19} - V_r^{22} + \pi V_r^{18} - \left(P_r^d + V_r^{21}\right)\right] = 0 \quad (32)$$

In each division r the difference between the value of water used in the farm and the value of water stored in the dam for reuse plus the value of the π portion of the runoff water used in the peak month in alleviating the peak flow constraint cannot exceed the cost of storing and pumping runoff water to or from the storage plus the value of the capacity of the storage. If the difference between the values of water used in the farm and stored in the dam plus the value of alleviating the peak flow constraint is less than the cost of storing then no water will be stored in the dam.

Value of peak water flow

$$V_1^{18} \leq V_1^{17} \text{ and }, q_{11}\left(V_1^{18} - V_1^{17}\right) = 0 \quad (33)$$

The value of peak water flow at node 1 reach 1 cannot exceed the value of the channel capacity constraint at node 1, and if the value at node 1 reach 1 is less than the value of the channel capacity constraint at source then no water flows in the peak month to reach 1.

$$V_{ir}^{18} \geq V_{jr'}^{18} \text{ and } Q^{jr'}\left(V_{ir}^{18} - V_{jr'}^{18}\right) = 0 \text{ for } \forall\, j \in j_i, r' \in r'_i \tag{34}$$

The value of peak water flow at node i and reach r cannot be less than the value of peak water flow at the next downstream node for any of the subsequent reaches, and if for any downstream reach the value is less than the value at the reach just upstream from it then no water flows to this downstream reach. Note that (33) and (34) imply that if water is used in the peak month at some downstream reach r, then the value of peak water flow at source, V_{ps}, is related to the value of peak water flow at this reach by $V_{ps} = V_1^{17} + V_{11}^{18} = V_{ir}^{18}$. Also note that (33) and (34) ignore seepage, escapes and evaporation losses and thus if there is water used at a downstream reach r, then the value of peak water flow at that reach the same as the value at all direct upstream reaches, *r'*, all the way to the source. This is different from the efficient pricing formulation in which the model conditions imply that at the optimum, in the presence of conveyance losses the value of peak flow water increases with distance from source until water flows cease.

Annual allocation water flow

$$V_1^{20} \leq V_1^{16} \text{ and } , Q_{11}\left(V_1^{20} - V_1^{16}\right) = 0 \tag{35}$$

The value of allocation water at node 1 reach 1 cannot exceed the value of allocation water at source, and if the value at node 1 reach 1 is less than the value at source then no allocation water flows to reach 1.

$$V_{ir}^{20} \geq V_{jr'}^{20} \text{ and } Q^{jr'}\left(V_{ir}^{20} - V_{jr'}^{20}\right) = 0 \text{ for } \forall\, j \in j_i, r' \in r'_i \tag{36}$$

The value of allocation water at node i and reach r cannot be less than the value of water at the next downstream node for any of the subsequent reaches, and if for any downstream reach the value is less than the value at the reach just upstream from it then no water flows to this downstream reach. Note that (35) and (36) imply that if water is used at some downstream reach r, then the value of allocation water at source, V_{as}, is related to the value of water at this reach by $V_{as} = V^{16} = V_{ir}^{20}$. Also note that, just as for the value of peak water flow, (35) and (36) ignore seepage, escapes and evaporation losses and thus if there is water used at a downstream reach r, then the value of allocation water at that reach the same as the value at all direct upstream reaches, *r'*, all the way to the source. This is again different from the efficient pricing formulation in which the model conditions imply that at the optimum, in the presence of conveyance losses the value of allocation water increases with distance from source until water flows cease.

$$V_r^{20} \geq V_r^{19} - P_r \text{ and } CW_r\left[V_r^{20} - \left(V_r^{19} - P_r\right)\right] = 0 \text{ for } \forall r \text{ and } m \tag{37}$$

At each reach *r*, the value of water cannot be less than the value of water at the farm group assigned to this reach less the delivery charge of water at the offtake and if this value exceeds the value of water used in the division then no canal water will be flowing to that farm group.

Solution of uniform pricing problem

Optimal values for the farmers' and water authorities' decisions subject to uniform water prices prevailing are obtained as the solution to the conditions (16)-(37). The solution is fully defined by these conditions and can be obtained in a number of ways. Here, the solution is obtained by maximising the objective function

$$\begin{aligned}
&\sum_{rbt} MIB_{bt} \times A_{rbt} + \sum_{r} MDB \times A_r^{dry} + \sum_{rht} MH_{ht} \times A_{rht} \\
&- DCST_r - P_r^d * DW_r - \sum_r CW_r P_r \\
&- V^1 D - V_1^{17} CK_1 - \sum_r DK_r V_r^{22} \\
&- \sum_r IBLND_r V_r^{23} - \sum_r RLIM_r V_r^{24} - \sum_r DBLND_r V_r^{25} \\
&- \sum_{r,h} HLND_r V_r^{26} \\
&+ \sum_{i,r} q_{ir}\left(\delta_r \mu_r V_r^{18}\right) + \sum_{i,r} Q_{ir}\left(\delta_r \mu_r V_r^{20}\right)
\end{aligned} \tag{38}$$

with respect to nonnegative price and volume variables subject to the inequality constraints for conditions (16)-(37). The objective function represents, for the whole irrigation system, the annual gross margin on all farm groups, less the total capital and operating cost of reusing runoff water, total delivery charge on the water entering all the farm groups, rent to water allocations, rent to channel capacity constraints and all annual land rents plus the value of all evaporation, seepage and escape losses. The last two terms in the criterion (38) are the sum over all nodes and reaches of the value of all evaporation, seepage and escape losses evaluated at the optimum uniform water price. These terms can also be interpreted as the sum - over nodes *i* and reaches (or farms) *r* - of the value of the *ad valorem* subsidy to a water user *r* at the rate $(\delta_r \mu_r)$

that is implicit in water charges set at a second best uniform price. Second best, in the sense that within the set of all possible uniform prices the optimal uniform price is obtained. Note: the implicit subsidy is expressed in terms of this second best optimum price not in terms of the price to the user that would prevail in the unrestricted optimum. In optimum, the value of the criterion must be zero.

Appendix A: Estimation of cost of refurbishment

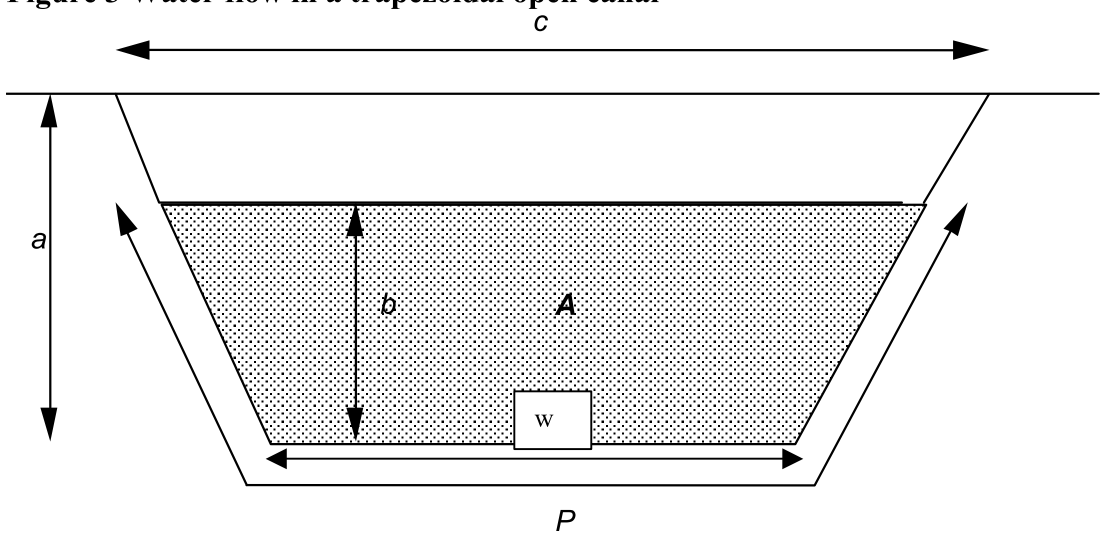
There is little information available on the relationship between the actual cost of irrigation infrastructure refurbishment and the design flow rate for most irrigation areas. As a result an engineering approach can be used to estimate the relationship between the cost of refurbishment per lineal metre and the design flow rate for each option.

Cost of refurbishment of an irrigation canal reach depends on the option chosen, the length of the reach and the flow rate required in the reach. The required flow rate in a given reach depends on the rate of withdrawal of water by the farms assigned to the reach and the downstream reaches and the rate of loss of water due to seepage, evaporation and escapes in downstream reaches.

The general approach to estimate a relationship between cost of refurbishment and design flow rate involves three steps. First, the hydraulic functional relationships between the flow rate and the canal or pipe geometry can be derived for a range of plausible design flow rates. Second, the specifications of canal or pipe geometry can be used to derive quantities of materials and labour required for refurbishment for a range of flow rates. Labour and machinery are required for breaking up and removing existing lined canals or pipes, and for earth works for reshaping or excavation. Material inputs include concrete, clay and plastic membrane for lining and new pipes. Third, using prevailing prices of materials and hired labour, the cost of refurbishment per lineal metre can be estimated for a range of flow rates for each of the options.

Open canals

Figure 3 Water flow in a trapezoidal open canal



The functional relationship between the rate of flow in an open trapezoidal canal and its geometry is given by the Manning's equation (equation 1).

$$Q = \frac{A.R^{2/3}.S^{1/2}}{n} \qquad (1)$$

Where $R = A/P$

Q	rate of flow (m^3/second)
A	cross sectional area of water flow (m^2)
R	hydraulic radius (m)
P	wetted perimeter (m)
S	slope of canal (eg. 1 in 100 = 0.01)
n	friction coefficient

The values for *A* and *P* can be estimated to cover the range of open canal specifications found in the study area. The flow rates can be estimated using equation 1. In the case of concrete and clay lining, the volume of material required for lining 1 lineal metre of canal to a specified thickness can then be estimated. The total cost of refurbishment of 1 lineal metre of canal including the cost of breaking up and removing existing linings, reshaping and excavation can be estimated for the range of flow rates assumed.

Gravity pipes

The rate of flow in a gravity fed pipe is a function of the diameter of the pipe and the friction head loss per metre. The Hazen-Williams formula provides for a practical way of relating flow rate to pipe diameter in gravity fed pipes.

$$V = 0.355Cd^{0.63}\left(\frac{H}{L}\right)^{0.54} \tag{2}$$

Where	V	velocity of flow (m/s)
	d	diameter (m)
	H	friction head loss (m)
	L	length of canal (m)
	C	a coefficient

The velocity V can also be defined in terms of flow rate Q, which is measured in m^3/s.

$$V = \frac{Q}{\pi(d/2)^2}$$

Substituting the value of V in equation 2 yields.

$$Q = 0.08875\pi C\left(\frac{H}{L}\right)^{0.54} d^{2.63} \tag{3}$$

The flow rates can be estimated for a range of diameters of pipes available from a supplier. The prices per lineal metre of pipes with different diameters can be obtained from CSR Humes.

A linear relationship between the cost of refurbishment per lineal metre and the flow rate can be estimated for each option (equation 4).

$$C_k = \alpha_k + \beta_k Q_k \tag{4}$$

Where C_k = cost of refurbishment with option k (\$/m)

Q_k = flow rate with refurbishment option k (ML/day); and

α_k and β_k are coefficients

Appendix B LP Tableau of the simple model

	LP Tableau -Reach 1																								
		Invest Conv Earth	Invest Conv Conc	Invest Conv Piped	Invest Dam	Dam water	Crop water	Vine furrow	Vine drip	Citrus furrow	Citrus drip	Rice tech1	Rice tech2	Wheat tech1	Wheat tech2	Land trnas	Dry cropin	Peak flow earth	Peak flow conc	Peak flow piped	Annual flow earth	Annual flow conc	Annual flow piped		RHS
	Objective function	$-\alpha^{e}.\mu_{r}$	$-\alpha^{c}.\mu_{r}$	$-\alpha^{p}.\mu_{r}$	$-ID_{r}$	$-P^{d}_{r}$	$-P_{r}$	MVF_{r}	MVD_{r}	MCF_{r}	MCD_{r}	MRT_{1r}	MRT_{1r}	MWT_{1}	MWT_{1r}		MDW_{r}	$-\beta^{e}.\mu_{r}$	$-\beta^{c}.\mu_{r}$	$-\beta^{p}.\mu_{r}$	$-P^{I}_{r}$	$-P^{I}_{r}$	$-P^{I}_{r}$	**Continued**	
R1	**Conveyance type**	1	1	1																				<=	1
R1	**Vine land**							1	1															<=	$HLND_{vr}$
R1	**Citrus land**									1	1													<=	$HLND_{or}$
R1	**Irrig land**											1	1	1	1	1								<=	$IBLND_{r}$
R1	**Rice lim**											1	1											<=	$RLIM_{r}$
R1	**Dry land**															-1	1							<=	$DBLND_{r}$
R1	**Chan capacity -e**	$-CK_{r}$																1						<=	0
R1	**Chan capacity -c**		$-CK_{r}$																1					<=	0
R1	**Chan capacity -p**			$-CK_{r}$																1				<=	0
R1	**Flow control -e**	$-F_{r}$																			1			<=	0
R1	**Flow control -c**		$-F_{r}$																			1		<=	0
R1	**Flow control -p**			$-F_{r}$																			1	<=	0
R1	**Divert -earth**																				1			<=	D
R1	**Divert -conc**																					1		<=	D
R1	**Divert -piped**																						1	<=	D
R1	**Run off**					1		$-\sigma^{vf}$	$-\sigma^{vd}$	σ^{cf}	σ^{cd}	σ^{r1}	σ^{r2}	σ^{w1}	σ^{w2}									<=	0
R1	**Crop water**						-1	$\varepsilon^{vf}/\theta^{vf}$	$\varepsilon^{vd}/\theta^{vd}$	$\varepsilon^{cf}/\theta^{cf}$	$\varepsilon^{cd}/\theta^{cd}$	$\varepsilon^{r1}/\theta^{r1}$	$\varepsilon^{r2}/\theta^{r2}$	$\varepsilon^{w1}/\theta^{w}$	$\varepsilon^{w2}/\theta^{w2}$										
R1	**Dam water**				$-DK_{r}$	1																		<=	0
R1	**Sys cont (annual)**					-1	1														$-(1-\delta^{e}.I_{r})$	$-(1-\delta^{c}.I_{r})$	$-(1-\delta^{p}.I_{r})$	**Continued**	
R1	**Sys cont (peak)**					$-\pi$		γ^{vf}/θ^{vf}	γ^{vd}/θ^{vd}	γ^{cf}/θ^{cf}	γ^{cd}/θ^{cd}	γ^{r1}/θ^{r1}	γ^{r2}/θ^{r2}					$-(1-\delta^{e}.I_{r})$	$-(1-\delta^{c}.I_{r})$	$-(1-\delta^{p}.I_{r})$				**to R2**	

LP Tableau -Reach 2

		Invest Conv Earth	Invest Conv Conc	Invest Conv Piped	Invest Dam	Dam water	Crop water	Vine furrow	Vine drip	Citrus furrow	Citrus drip	Rice tech1	Rice tech2	Wheat tech1	Wheat tech2	Land trnas	Dryland croppin	Peak flow earth	Peak flow conc	Peak flow piped	Annual flow earth	Annual flow conc	Annual flow piped		RHS
	Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$	$-ID_r$	$-P^d_r$	$-P_r$	MVF_r	MVD_r	MCF_r	MCD_r	MRT_{1r}	MRT_{1r}	MWT_1	MWT_{1r}		MDW_r	$-\beta^e.\mu_r$	$-\beta^c.\mu_r$	$-\beta^p.\mu_r$	$-P^I_r$	$-P^I_r$	$-P^I_r$	Continued	
R1	Sys cont (annual) (R1)																				1	1	1	<=	0
R1	Sys cont (peak) (R1)																	1	1	1				<=	0
R2	Conveyance type	1	1	1																				<=	1
R2	Vine land							1	1															<=	$HLND_{vr}$
R2	Citrus land									1	1													<=	$HLND_{or}$
R2	Irrig land											1	1	1	1	1								<=	$IBLND_r$
R2	Rice lim											1	1											<=	$RLIM_r$
R2	Dry land															-1	1							<=	$DBLND_r$
R2	Chan capacity -e	$-CK_r$																1						<=	0
R2	Chan capacity -c		$-CK_r$																1					<=	0
R2	Chan capacity -p			$-CK_r$																1				<=	0
R2	Flow control -e	$-F_r$																			1			<=	0
R2	Flow control -c		$-F_r$																			1		<=	0
R2	Flow control -p			$-F_r$																			1	<=	0
R2	Run off					1		σ^{vf}	σ^{vd}	σ^{cf}	σ^{cd}	σ^{r1}	σ^{r2}	σ^{w1}	σ^{w2}									<=	0
R2	Crop water						-1	$\varepsilon^{vf}/\theta^{vf}$	$\varepsilon^{vd}/\theta^{vd}$	$\varepsilon^{cf}/\theta^{cf}$	$\varepsilon^{cd}/\theta^{cd}$	$\varepsilon^{r1}/\theta^{r1}$	$\varepsilon^{r2}/\theta^{r2}$	$\varepsilon^{w1}/\theta^{w}$	$\varepsilon^{w2}/\theta^{w2}$										
R2	Dam water				$-DK_r$	1																		<=	0
R2	Sys cont (annual)					-1	1														$-(1-\delta^e.l_r)$	$-(1-\delta^c.l_r)$	$-(1-\delta^p.l_r)$	Continued	
R2	Sys cont (peak)					$-\pi$		γ^{vf}/θ^{vf}	γ^{vd}/θ^{vd}	γ^{cf}/θ^{cf}	γ^{cd}/θ^{cd}	γ^{r1}/θ^{r1}	γ^{r2}/θ^{r2}					$-(1-\delta^e.l_r)$	$-(1-\delta^c.l_r)$	$-(1-\delta^p.l_r)$				to R3	

LP Tableau -the tail reach (Reach 5)

		Invest Conv Earth	Invest Conv Conc	Invest Conv Piped	Invest Dam	Dam water	Crop water	Vine furrow	Vine drip	Citrus furrow	Citrus drip	Rice tech1	Rice tech2	Wheat tech1	Wheat tech2	Land trnas	Dryland cropping	Peak flow earth	Peak flow conc	Peak flow piped	Annual flow earth	Annual flow conc	Annual flow piped		RHS
	Objective function	$-\alpha^e.\mu_r$	$-\alpha^c.\mu_r$	$-\alpha^p.\mu_r$	$-ID_r$	$-P^d_r$	$-P_r$	MVF_r	MVD_r	MCF_r	MCD_r	MRT_{1r}	MRT_{1r}	MWT_1	MWT_{1r}		MDW_r	$-\beta^e.\mu_r$	$-\beta^c.\mu_r$	$-\beta^p.\mu_r$	$-P^I_r$	$-P^I_r$	$-P^I_r$		
R4	**Sys cont (annual) (R1)**																				**1**	**1**	**1**	<=	0
R4	**Sys cont (peak) (R1)**																	**1**	**1**	**1**				<=	0
R5	**Conveyance type**	1	1	1																				<=	1
R5	**Vine land**							1	1															<=	$HLND_{vr}$
R5	**Citrus land**									1	1													<=	$HLND_{or}$
R5	**Irrig land**											1	1	1	1	1								<=	$IBLND_r$
R5	**Rice lim**											1	1											<=	$RLIM_r$
R5	**Dry land**															-1	1							<=	$DBLND_r$
R5	**Chan capacity -e**	$-CK_r$																1						<=	0
R5	**Chan capacity -c**		$-CK_r$																1					<=	0
R5	**Chan capacity -p**			$-CK_r$																1				<=	0
R5	**Flow control -e**	$-F_r$																			1			<=	0
R5	**Flow control -c**		$-F_r$																			1		<=	0
R5	**Flow control -p**			$-F_r$																			1	<=	0
R5	**Run off**					1		σ^{vf}	σ^{vd}	σ^{cf}	σ^{cd}	σ^{r1}	σ^{r2}	σ^{w1}	σ^{w2}									<=	0
R5	**Crop water**						-1	$\varepsilon^{vf}/\theta^{vf}$	$\varepsilon^{vd}/\theta^{vd}$	$\varepsilon^{cf}/\theta^{cf}$	$\varepsilon^{cd}/\theta^{cd}$	$\varepsilon^{r1}/\theta^{r1}$	$\varepsilon^{r2}/\theta^{r2}$	$\varepsilon^{w1}/\theta^{w}$	$\varepsilon^{w2}/\theta^{w2}$										
R5	**Dam water**				$-DK_r$	1																		<=	0
R5	**Sys cont (annual)**					-1	1														$-(1-\delta^e.I_r)$	$-(1-\delta^c.I_r)$	$-(1-\delta^p.I_r)$	=<=	0
R5	**Sys cont (peak)**					$-\pi$		γ^{vf}/θ^{vf}	γ^{vd}/θ^{vd}	γ^{cf}/θ^{cf}	γ^{cd}/θ^{cd}	γ^{r1}/θ^{r1}	γ^{r2}/θ^{r2}					$-(1-\delta^e.I_r)$	$-(1-\delta^c.I_r)$	$-(1-\delta^p.I_r)$				=<=	0